

SELF COMPLEMENTARY BROADBAND ANTENNA STRUCTURES

Self-complementary structures, Babinet's principle, and Mushiake's work form a foundational framework for designing broadband and frequency-independent antennas.

Babinet's Principle in Electromagnetics and Antennas

Babinet's principle originated in optics: the diffraction pattern from an opaque screen with apertures equals (in a complementary way) the pattern from the complementary screen (opaque where there were holes, and transparent where there was opaque material). The fields from the screen and its complement add up to the unobstructed field.

In antenna theory, it applies to complementary planar structures (e.g., a thin wire dipole antenna and its complementary slot antenna cut into an infinite conducting sheet). The key relation from Babinet's principle for impedances is that the input impedance of an antenna Z_1 and its complementary structure Z_2 satisfy: $Z_1 \cdot Z_2 = \left(\frac{\eta}{2}\right)^2$ where $\eta \approx 377 \Omega$ is the intrinsic impedance of free space ($Z_0 = \sqrt{\mu_0/\epsilon_0}$). This product is constant. [Reddit +1](#)

This duality links electric (dipole-like) and magnetic (slot-like) responses and is extremely useful for analysis and design.

Self-Complementary Structures

A **self-complementary antenna (SCA)** is a structure where the conducting parts and the non-conducting (slot/air) parts are geometrically identical to each other—meaning the antenna exactly overlays its own complement after rotation and/or translation.

Examples include certain spiral shapes (especially equiangular spirals), "alternate-leaves" or square-type designs, bow-tie variants, and more complex multi-terminal or 3D versions. In the ideal (infinitely extending) case, the geometry has infinite varieties but enforces self-duality. [Wikipedia](#)

Because the structure *is* its own complement, the impedance relation simplifies dramatically to a constant value independent of frequency and (within broad classes) independent of specific shape details.

Mushiake's Discovery and the Mushiake Relationship

In 1948, **Yasuto Mushiake** (Tohoku University, Japan) discovered the **Principle of Self-Complementarity** while investigating the input impedances of slit/slot antennas (building on earlier Japanese work on Babinet's principle in EM fields and slot antennas, and his own Yagi-Uda studies). [Etw](#)

He originated the self-complementary geometry and proved its constant-impedance property. This is recognized as an **IEEE Milestone** (dedicated at Tohoku University).

The specific formula for the simplest two-terminal planar self-complementary antenna is known as the **Mushiake Relationship** (sometimes called Mushiake's relation): $Z = \frac{Z_0}{2} \approx 188.4 \Omega$ (where $Z_0 \approx 377 \Omega$).

[Wikipedia](#)

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Mushiake and collaborators later generalized it to multi-terminal, 3D, stacked, and other complexities (e.g., four-terminal turnstile-type yields different constant values like $\sim 133 \Omega$ per terminal in certain excitations). The principle holds for classes of structures with varying complexity in terminals and reference planes.

Mushiake authored key papers and a book on the topic (*Self-Complementary Antennas: Principle of Self-Complementarity for Constant Impedance*, 1996). Victor Rumsey and others in the US later referenced and built upon it in frequency-independent antenna work. [IEEE Xplore](#)

Significance for Broadband Antennas

- **Constant impedance** eliminates frequency-dependent mismatch losses, enabling very wide bandwidths (theoretically unlimited in the ideal infinite case; practically limited by truncation, feed region, and size).
- When combined with **self-similarity** (scaling properties) or designs where the "active region" shifts with frequency (as in spirals or log-periodics), the radiation pattern and gain can also remain stable.
- Finite/truncated versions (e.g., "square SCA" or alternate-leaves) still deliver excellent broadband performance with nearly omnidirectional patterns in some designs. [Wikipedia](#)

Examples and applications:

- **Spiral antennas** (equiangular or Archimedean variants) — classic frequency-independent designs.
- **Log-periodic dipole arrays (LPDAs)** — often described as modified or "folded" self-complementary structures with transposed excitation for unidirectional patterns. The self-complementarity provides the underlying broadband impedance stability (not the log-periodic shape alone). [Wikipedia](#)
- Bow-tie, certain fractal-like, and planar monopole/notch combinations.
- Used in TV reception (since the 1950s), wireless broadband, radio astronomy, cellular/UWB, 5G MIMO, and more.

Mushiake emphasized that self-complementarity provides the rigorous constant-impedance foundation, while practical designs often modify it slightly for gain, directivity, or size.

In summary, Babinet's principle provides the duality, self-complementary geometry exploits it for invariance, and Mushiake's 1948 insight turned it into a powerful, general design principle that underpins many of today's ultra-wideband antennas. The beauty is its simplicity: check if a shape is self-complementary, and you immediately know a key performance parameter (impedance) without heavy computation.

Common examples include equiangular spiral antennas (highly self-complementary) and square/alternate-leaves planar designs.